FP2 Matrices

- minors – determinant excluding row & column of an element
  eg minor of 2 from $\begin{bmatrix} 2 & -1 & 4 \\ 0 & 3 & -2 \\ -4 & 1 & -3 \end{bmatrix}$ is $\begin{bmatrix} 3 & -2 \\ 1 & -3 \end{bmatrix}$

- cofactors are signed $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

  eg cofactor of -1 in

  $\begin{bmatrix} 2 & -1 & 4 \\ 0 & 3 & -2 \\ -4 & 1 & -3 \end{bmatrix}$ is $-\begin{vmatrix} 0 & -2 \\ -4 & -3 \end{vmatrix} = -8$

- determinant ΔA, det A or $|A| = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$
  (say expanded by first column A_{rc} are)

  eg $\begin{vmatrix} 2 & -1 & 4 \\ 0 & 3 & -2 \\ -4 & 1 & -3 \end{vmatrix} = (2)\begin{vmatrix} 3 & -2 \\ 1 & -3 \end{vmatrix} - (0)\begin{vmatrix} -1 & 4 \\ 1 & -3 \end{vmatrix} + (-4)\begin{vmatrix} -1 & 4 \\ 3 & -2 \end{vmatrix}$

  $= 2(-7) - 4(-10)$

  $= 26$

  Remember to use cofactor signs
  eg expanding by first row
  det A above $= 2(-7) - (-1)(-8) + 4(12) = 26$

- An eigenvector of a matrix M is a vector $\underline{s}$ in the direction of an invariant line through the origin. When $\underline{s}$ is multiplied by M the result is a vector in the same direction with possibly a different magnitude

  $\therefore M\underline{s} = \lambda\underline{s}$ where $\underline{s}$ is the eigenvector, and λ the eigenvalue.

- There could be many eigenvectors (equivalent) corresponding to an eigenvalue λ.
- use $\det(M - \lambda I) = 0$ to find the characteristic equation and solve for λ to find the eigenvalues.

- $M^n s = \lambda^n s$ let $M\underline{s} = \lambda\underline{s}$

  then $MM\underline{s} = M\lambda\underline{s} = \lambda M\underline{s} = \lambda\lambda\underline{s} = \lambda^2\underline{s}$

  $\therefore M^2\underline{s} = \lambda^2\underline{s}$

  $MM^2\underline{s} = M\lambda^2\underline{s} = \lambda^2 M\underline{s} = \lambda^2\lambda\underline{s} = \lambda^3\underline{s}$

  $\therefore M^3\underline{s} = \lambda^3\underline{s}$ etc.

- If $\det M = 0$ then cannot find inverse similarly from FP1 if $\det M \neq 0$ then M^{-1} exists for a unique solution

  If $\det M = 0$ then
  - inconsistent & no unique solution
  - consistent & infinitely many solutions

- '3 equations' is the same as '3 planes' in space

inconsistent
- could be 3 parallel planes (or 2 coincident & 1 parallel) $\equiv$
- 2 planes intersect with a 3rd plane 
- 3 planes form a triangular prism  (eg toblerone)

} solutions common to 2

consistent
- 3 planes are coincident
- 3 planes have a common line  (sheaf)

} infinite solutions common to all 3

- Diagonalise a matrix M using $M^n = S\Lambda^n S^{-1}$

  where S matrix of eigenvectors in columns (v_1, v_2, v_3)

  Λ diagonal matrix of eigenvalues $\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$

  Rearranging can give

  $\Lambda^n = S^{-1}M^n S$ if they ask for a diagonal matrix.

- The magnitude of a matrix could be derived from $M^n\underline{s} = \lambda^n\underline{s}$ especially if $M^3\underline{s} = \lambda^3\underline{s}$ then magnitude is just λ^3 (it is after all just a scalar quantity)

- To show a vector is an eigenvector use $M\underline{s} = \lambda\underline{s}$ and multiply out $M\underline{s}$ and factorise out $\underline{s}$ to find λ

  eg $M = \begin{pmatrix} 0 & -1 & 1 \\ 6 & -2 & 6 \\ 4 & 1 & 3 \end{pmatrix}$ $\underline{s} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ $\therefore M\underline{s} = \begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix} = -2\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ $\therefore \lambda = -2$.

  If $M\underline{s} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ remember you can take out $\lambda = 0$ as a factor of $\underline{s}$

- $\det(MN) = \det M \times \det N$ if both are $n \times n$

- $(MN)^{-1} = N^{-1}M^{-1}$ if both $n \times n$ and non singular

- $(MN)^T = N^T M^T$ if $M = m \times n$, N is $n \times p$

- transpose ^{T} is $M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \Rightarrow M^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ rows become columns.