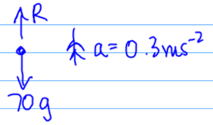
Mechanics 1 January 2008

Q1



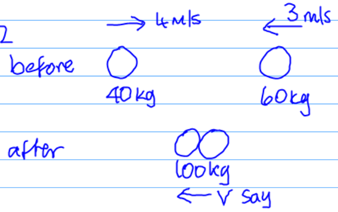
Using $F = ma$ with positive direction as upwards

$R - 70g = 70 \times 0.3$

$\therefore R = 686 + 21$

$\therefore R = 707$ N (3sf)

Q2



Using conservation of momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

Watch directions

$40 \times (4) + 60 \times (-3) = 100(-v)$

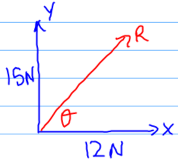
$\therefore 160 - 180 = -100v$

$\therefore 100v = 20$

$v = 0.2$ m/s

speed is 0.2 is direction that 60kg person originally moving in

Q3.



$\Longrightarrow$

(i) $\therefore R^2 = 15^2 + 12^2$

$R^2 = 225 + 144$

$R = 19.21 \text{ N}$

$\therefore R = 19.2 \text{N}$ (3sf)

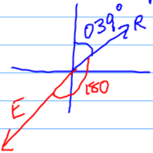
$\tan\theta = \frac{15}{12}$ $\quad \therefore \theta = \tan^{-1}\left(\frac{15}{12}\right) = 51.34^\circ$

$\therefore$ bearing is $90 - \theta = 038.7^\circ$

brg is 039° (3sf)

(ii) E keeps particle in equilibrium

$\therefore$ E is equal and opposite to R



E is 19.2N on a bearing of

$180 + 39 = 219^\circ$ (3sf)

Q4 $x = t^4 - 8t^2 + 16 \quad t \geq 0$

(i) when $t=0$ is particle at rest at point O

check $x=0$ when $t=2$ $\therefore x = 2^4 - 8\times 2^2 + 16$

$\therefore x = 16 - 32 + 16$

$\therefore x = 0$ ie at O when $t=2$.

check $v=0$ when $t=2$

$v = \frac{dx}{dt} = 4t^3 - 16t$

When $t=2$ $v = 4\times 8 - 16\times 2$

$\therefore v = 32 - 32 = 0$

$\therefore$ V at rest when $t=2$

Can also factorise $v = 4t(t^2-4) = 4t(t-2)(t+2)$

$\therefore$ when $v=0$, $t=0$, $t=\pm 2$.

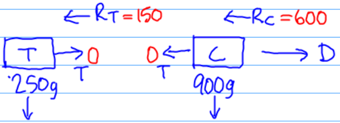
Q 4(ii) $v = 4t^3 - 16t$

$\therefore a = 12t^2 - 16$

When $t = 2$ $a = 12 \times 4 - 16 = 48 - 16$

$\therefore a = 32 \text{ m/s}^2$

Q5 (i)



(a) using 'F=ma' on the trailer alone

$$T - R_T = ma$$
$$0 - 150 = 250a$$
$$a = \frac{-150}{250} = -0.6 \text{ m/s}^2$$

(b) using 'F=ma' on the car alone

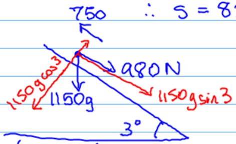
use the acceleration you found for trailer (it will be the same)

$$D - R_C - T = ma$$
$$D - 600 - 0 = 900 \times (-0.6)$$
$$D = 600 - 540$$
$$\therefore D = 60 \text{ N.}$$

5(i)(c) to reduce from 18 m/s to 15 m/s when $a = -0.6$

$$v^2 = u^2 + 2as$$
$$15^2 = 18^2 + 2(-0.6)s$$
$$\therefore s = \frac{18^2 - 15^2}{1.2}$$
$$\therefore s = 82.5\text{ m} \quad (3sf)$$

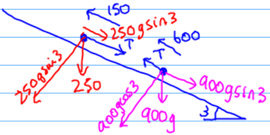
(ii)



Consider complete system because we don't know T for this case.
driving force $D = 980$ downwards
total resistance $R = 600 + 150$ upwards

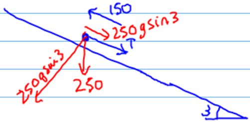
Using 'F = ma' down the slope:
Coming down the slope

$$D - R + 1150g\sin(3) = 1150a$$
$$980 - 750 + 1150 \times 9.8 \times \sin(3) = 1150a$$
$$\therefore a = 0.713\text{ ms}^{-2}. \quad (3sf)$$



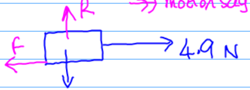
We can work a value for T since we have 'a': we can separate into car or trailer components

(ib) Consider the trailer alone $250g\sin(3)+T-150 = 250a$
with 'a' you just worked out. $\therefore T=199.99 = 200\text{ N}$ (3sf)



Q6

(i)



note you draw own sketch for (i)

Mass $= \frac{14.7}{9.8}$ kg

using limiting equilibrium $(a=0)$

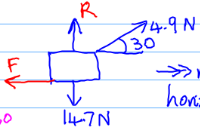
$R = 14.7$

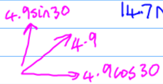
$F = 4.9$

$\therefore\ F = \mu R$

$\mu = \frac{F}{R} = \frac{4.9}{14.7} = \frac{1}{3}$

(ii)





→ motion using 'F=ma'

horizontal $4.9\cos 30 - F = \frac{14.7}{9.8}a$

consider vertical $R + 4.9\sin 30 = 14.7$

$\therefore R = 14.7 - 2.45$

$\therefore R = 12.25$ N

so with $F = \frac{1}{3}R$ $F = 4.08\dot{3}$

$\therefore$ $R = 12.3$ and $F = 4.08$ N (3sf)

6 (ii) (b) acceleration of block using 'F = ma'

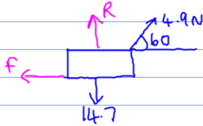
$$4.9\cos 30 - F = \frac{14.7}{9.8} a$$

using $F = 4.08\dot{3}$ from before

$$\therefore a = 0.1068 \text{ ms}^{-2}$$

$$\therefore a = 0.107 \text{ ms}^{-2} \text{ (3sf)}$$

(iii)



$$R + 4.9\sin 60 = 14.7$$

$$\therefore R = 14.7 - 4.9\sin 60$$

$$\therefore R = 10.456 \text{ N}$$

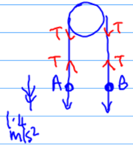
$$F = \mu R = \frac{1}{3} R = 3.4855 \text{ N}$$

but 'F = ma' gives $4.9\cos 60 - F = \frac{14.7}{9.8} a$

$$\therefore 2.45 - 3.4855 = \frac{14.7}{9.8} a$$

Since $\mu R > 2.45$ we do not have limiting equilibrium so $F = 2.45$ N.

Q7.



$t = 0.8$ before A hits floor.

(i) $u = 0$ $\quad S = ut + \frac{1}{2}at^2$

$v = ?$ $\quad S = 0 + \frac{1}{2} \times 1.4 \times 0.8^2$

$a = 1.4$

$s = ?$ $\quad \therefore S = 0.448$ m.

$t = 0.8$

$v = u + at$

$v = 0 + 1.4 \times 0.8$

$\therefore v = 1.12$ m/s

(ii) B continues upwards but now $a = -9.8$ m/s² (not 1.4)

B has speed of A until A hits the floor; B then starts slowing down. NEW ACCELERATION is gravity.

$u = 1.12$

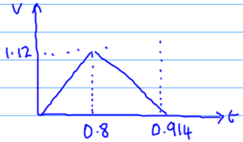
$v = 0$ $\quad \therefore v^2 = u^2 + 2as$ $\quad v = u + at$

$s = ?$ $\quad 0 = 1.12^2 + 2(-9.8)s$ $\quad 0 = 1.12 + (-9.8)t$

$\therefore s = \frac{1.12^2}{19.6} = 0.064$ m $\quad \therefore t = \frac{1.12}{9.8} = 0.114$

$\therefore$ B in motion for $0.8 + 0.114 = 0.914$ s.

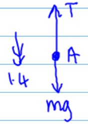
(iii)



mass of A

(iv) Tension at A = 5.88 N

using 'F=ma' down

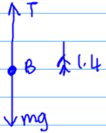


$mg - T = m \times 1.4$

$\therefore m(9.8 - 1.4) = 5.88$

$\therefore m = \frac{5.88}{8.4} = 0.7\,\text{kg}$

mass of B



'F=ma' upwards

$T - mg = ma$

$T = mg + ma$

$5.88 = m(9.8 + 1.4)$

$m = \frac{5.88}{11.2} = 0.525\,\text{kg}$

7(v) Pulley mass 0.5kg

(a)

before A hits the floor, $T = 5.88$ N (given)

$$\therefore R = 2T + 0.5g$$
$$= 2 \times 5.88 + 0.5 \times 9.8$$
$$= 16.7\text{ N} \quad (3\text{sf})$$

(b)

When A strikes the floor TENSION is ZERO
– string goes slack

$$\therefore R = 0.5g = 4.9\text{ N}$$