M1. January 2011

1.

$u = 2.4 \rightarrow$ $\qquad$ $\leftarrow u = -1.5$

P $\qquad$ Q

$m = 0.5$ $\qquad$ $m = 0.8$

$v = 0.2 \leftarrow$ $\qquad$ $\rightarrow v_2$

(i) change in momentum for P $= m(v-u) = 0.5\,(0.2 - -2.4)$

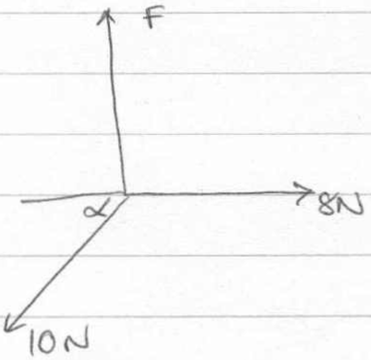
$= 1.3$ Ns.

(ii) Conservation of momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

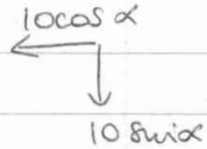
$\therefore\ 0.5\,(2.4) + 0.8\,(-1.5) = 0.5\,(-0.2) + 0.8\,(v_2)$

$\therefore\ v_2 = \frac{1}{8} = 0.125$ m/s in its original direction.

2.



(i) Resolving 10N into



$\therefore\ F = 10\sin\alpha$

& $8 = 10\cos\alpha$

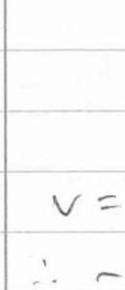
$\therefore\ \cos\alpha = 0.8$

$\therefore\ \alpha = 36.9°$

(ii) Since $F = 10\sin\alpha = 10\sin(\cos^{-1}0.8)$

$\therefore\ F = 6$ N.

3.



$u = 5$ m/s.

$v = ?$

$a = -9.8$

$s = -2.5$

$t =$

(i) displacement of ground is -2.5 metres

$\therefore\ v^2 = u^2 + 2as$

$v^2 = 25 - 19.6\,(-2.5)$

$v^2 = 74$

$\therefore\ v = -8.60$ m/s $\qquad$ (3sf)

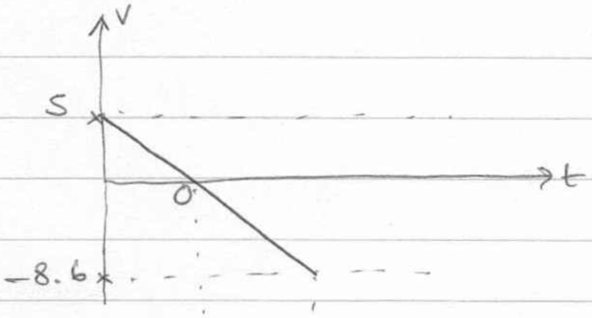
(negative velocity as going down)

$\therefore$ speed with which hits the ground is 8.60 m/s

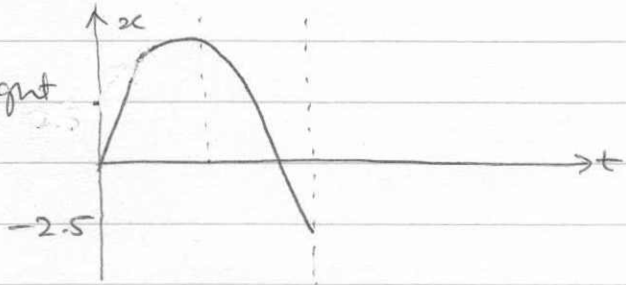
(ii) $v = u + at$

$\therefore\ -8.6 = 5 - 9.8t$ $\qquad$ $\therefore\ t = \frac{13.6}{9.8} = 1.39$ seconds (3sf)

(iii) (a) t-v graph

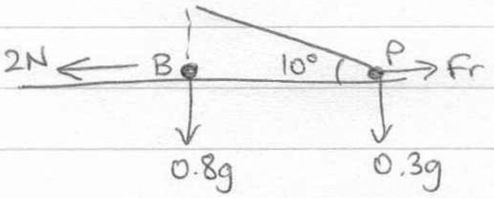


(b) t-x graph.
Where $v=0$ max height



starts at $x=0$
in my calcs
at end $x=-2.5$

4.



$a = 0.2 \ m/s^2$ friction at P, not B

(i) "$F=ma$" for whole system $\therefore 2 - F_r = 1.1 \times 0.2$

$m = 0.8 + 0.3 = 1.1$ $\therefore Fr = 2 - 1.1 \times 0.2$

$Fr = 1.78$

B: $T\cos 10$, T, $T\sin 10$

P: T, $T\sin 10$, $T\cos 10$

At B then "$F=ma$" $2 - T\cos 10 = 0.8 \times 0.2$

$\therefore T = \frac{1.84}{\cos 10} = 1.868\ldots$

$\therefore T = 1.87 \ N$ (3sf)

(ii) Coefficient of friction μ $F_r \le \mu R$ so $\mu = \frac{Fr}{R}$

at P : $F_r = 1.78$ N.

also R, $T\sin 10$, P, $0.3g$

Resolving vertically

$R + T\sin 10 = 0.3g$

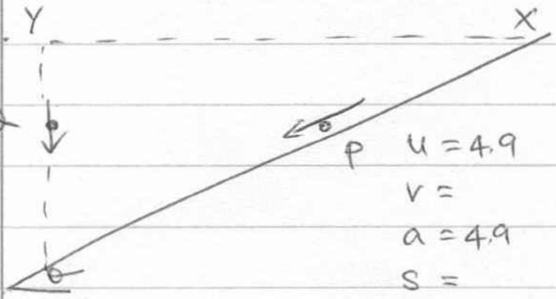
$\therefore R = 2.615\ldots$

$\therefore R = 2.62$ N (3sf)

$\therefore \mu = \frac{fr}{R} = \frac{1.78}{2.615\ldots} = 0.6805\ldots$

$\therefore \mu = 0.681$ (3sf)

Q5.



$u=0$, $v=$, $a=9.8$, $s=$, $t=T$

$u=4.9$, $v=$, $a=4.9$, $s=$, $t=T$

(i) for P using $s=ut+\frac{1}{2}at^2$

(a) $S_p = 4.9T + \frac{1}{2}\times 4.9 \times T^2$

for Q using $s=ut+\frac{1}{2}at^2$

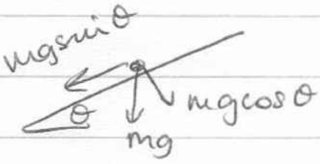
$S_q = 0 + \frac{1}{2}\times 9.8 \times T^2$

(b) using $f=ma$ on particle P

$mg\sin\theta = m\times 4.9$

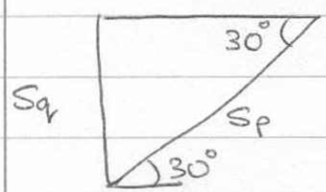
$\therefore \sin\theta = \frac{4.9}{9.8} = \frac{1}{2}$

$\therefore \theta = 30^\circ$



(smooth slope no friction)

(c)



$\sin 30 = \frac{S_q}{S_p}$ $\therefore S_p = \frac{S_q}{\sin 30} = 2S_q$

$\therefore S_p = 4.9T + \frac{1}{2}\times 4.9T^2 = 2S_q = 2\times\frac{1}{2}\times 9.8\times T^2$

$\therefore 4.9T + 2.45T^2 = 9.8T^2$

$\therefore 7.35T^2 - 4.9T = 0$

$\therefore T(7.35T - 4.9) = 0$

$\therefore T=0$ or $T = \frac{4.9}{7.35} = \frac{2}{3}$ seconds

(ii) speeds of particles before collision, using $v=u+at$

$V_p = 4.9 + 4.9\times\frac{2}{3} = 8.17$ m/s (3sf)

$V_q = 0 + 9.8\times\frac{2}{3} = 6.53$ m/s (3sf).

Q6. $v = t^2 - 9$ passes through O when $t=2$, (ie $x=0$, $t=2$)

(i) find x when $t=0$

$x = \int(t^2-9)\,dt$ since $v = \frac{dx}{dt}$

$\therefore x = \frac{t^3}{3} - 9t + c$

since at $x=0$, $t=2$ then $0 = \frac{8}{3} - 18 + c$ so $c = \frac{46}{3}$

So the displacement when $t=0$

using $x = \frac{t^3}{3} - 9t + \frac{46}{3}$

when $t=0$ $x = \frac{46}{3}$

(ii) distance travelled from $t=0$ to change of direction

displacement changes direction when $\frac{dx}{dt} = 0$ ie $v=0$

so $t^2 - 9 = 0$

$\therefore t = \pm 3$ (change cannot have negative t though)

using $t=3$ in $x = \frac{t^3}{3} - 9t + \frac{46}{3}$

$\therefore x = 9 - 27 + \frac{46}{3}$

$\therefore x = -\frac{8}{3}$

Since when $t=0$ $x = \frac{46}{3}$ and $t=3$ $x = -\frac{8}{3}$

distance $= \frac{46}{3} - \left(-\frac{8}{3}\right) = \frac{54}{3} = 18$

$\therefore$ distance travelled is 18 metres.

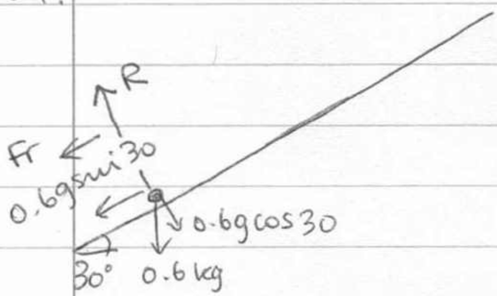
(iii) Distance of particle from O when $a = 10$ m/s^2

if $v = t^2 - 9$

$a = \frac{dv}{dt} = 2t$ if $a = 10$, $2t = 10$ so $t = 5$ secs.

$\therefore x = \frac{5^3}{3} - 45 + \frac{46}{3} = 12$ metres.

Q7.



(i) $a = -10$ m/s^2

using $F = ma$

$-Fr - 0.6g\sin 30 = 0.6 \times (-10)$

$\therefore Fr = 6 - 0.6g\sin 30 = 3.06$ N

since $R = 0.6g\cos 30$

and limiting friction $F \leq \mu R$

$\mu = \frac{Fr}{R} = \frac{3.06}{5.09\ldots} = 0.6009\ldots = 0.601$ (3sf)

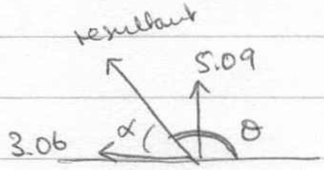
(ii) The contact force is the resultant of the normal reaction force R and the frictional force fr

(a) when in motion (as in (i))

$F_R = 3.06$ and $R = 5.09\ldots$

$\therefore$ resultant^2 = $3.06^2 + 5.09\ldots^2$ = 35.2944

$\therefore$ resultant = 5.94 N

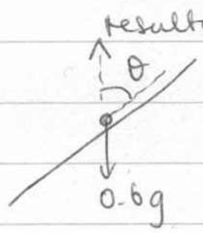


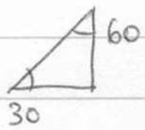
$\theta = 180^\circ - \tan\alpha$

$\tan\alpha = \dfrac{5.09\ldots}{3.06}$ $\alpha = 59.0^\circ$ (3sf)

$\therefore \theta = 121^\circ$ (3sf.)

(b) when at rest then friction changes direction - not needed now and the resultant force is just in the vertical line opposite to the mass / weight in downward direction





so resultant = 0.6×9.8 = 5.88 N.

$\theta = 60^\circ$ (vertically opp angles).