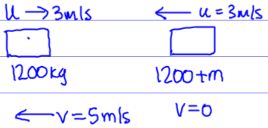
Mechanics 1

June 2006

Q1



You have to assume that the unloaded wagon changes direction after the collision.

Using conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

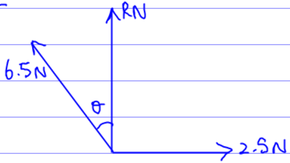
* watch change of signs for opposite velocities

$$1200 \times 3 - 3(1200 + m) = 1200 \times (-5) + 0$$

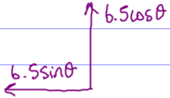
$$\therefore 3m = 6000$$

$$\therefore m = 2000 \text{ kg}$$

Q2



(i) Resolve the 6.5 N force into vertical & horizontal components



Equate horizontal components

$$6.5 \sin\theta = 2.5$$

$$\therefore \quad \theta = \sin^{-1}\left(\frac{2.5}{6.5}\right) = 22.6^\circ \quad (3sf)$$

(ii) Equate in vertical direction

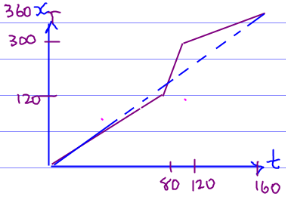
$$R = 6.5\cos\theta \quad = 6 \text{ N}.$$

Alternatively as a right angle triangle $R^2 + 2.5^2 = 6.5^2$

Q3. (i) walks 120 m at 1.5 m/s $\therefore t = \frac{120}{1.5} = 80$ seconds

runs 180 m at 4.5 m/s $\therefore t = \frac{180}{4.5} = 40$ seconds

walks 60 m at 1.5 m/s $\therefore t = \frac{60}{1.5} = 40$ seconds



line segments with same speed are parallel - same gradient

(ii) the woman jogs the 360 m at constant speed in same time

so $\frac{360}{160} = 2.25$ m/s

(iii) overtakes at intersection of lines

could do 2 ways (but not reading off graph for full marks)

woman: distance $= 2.25t$

man $s = 120 + 4.5(t - 80)$ for 2nd segment.

equate and $t = 106\frac{2}{3}$ seconds

Q4. $V = 2\text{m/s}$ for $0 \le t \le 10$ note constant speed

$v = 0.03t^2 - 0.3t + 2$ for $t \geq 10$

(i) displacement when $t = 10$

use constant speed $= \dfrac{\text{distance}}{\text{time}}$ $\therefore$ $2 = \dfrac{x}{10}$ so $x = 20$ metres

(ii) when $t \geq 10$ displacement by integrating v (must use $+c$)

$$x = 0.03\frac{t^3}{3} - 0.3\frac{t^2}{2} + 2t + c$$

when $t = 10$ we know $x = 20$ (i) so subs into above for c *

$\therefore x = 0.1 \times 1000 - 0.15 \times 100 + 2 \times 10 + c = 20$

$\therefore c = 5$

$\therefore x = 0.01t^3 - 0.15t^2 + 2t + 5.$

show where the conditions to work out C come from.

(iii) acceleration = 0.6 m/s^2 $\quad a = \dfrac{dv}{dt}$

$\therefore \dfrac{dv}{dt} = 0.06t - 0.3$

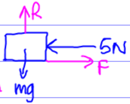
equate $\therefore$ $0.06t - 0.3 = 0.6$

$\therefore t = \dfrac{0.9}{0.06} = 15$ seconds

so when $t = 15$ s, $x = 0.01(15)^3 - 0.15(15)^2 + 2(15) + 5$

$\therefore x = 35$ metres from O.

Q5 (i) Note diagram relates to (ii)



$\mu = 0.2$ and $F \leq \mu R$ for limiting friction

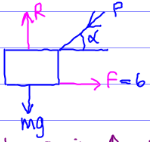
$R = mg$ and $F = \mu mg = 0.2mg$

However because of equilibrium $F = 5$ N

$\therefore 0.2mg = 5$ so $m = 2.551$ kg

$mg = 25$ useful later

(ii)

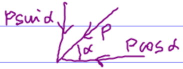


note P is pushing down on the block

$R = P\sin\alpha + mg$ for vertical components

and $F = P\cos\alpha$ for horizontal components

resolve P into ↕ and ↔



However $F = 6$ N so $P\cos\alpha = 6$ (1)

and $F = \mu R$ gives $0.2(P\sin\alpha + mg) = 6$

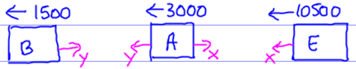
$\therefore P\sin\alpha + mg = 30$

But we know $mg = 25$ (i) above $\therefore P\sin\alpha = 5$ (2)

From (1) and (2) $\dfrac{P\sin\alpha}{P\cos\alpha} = \dfrac{5}{6}$ $\therefore \alpha = \tan^{-1}\left(\dfrac{5}{6}\right) = 39.8^\circ$ (3sf)

$\therefore P = 7.81$ N

Q6



Total mass = 80000 kg

⟹ motion

(i) Say the force driving the train is D, then using $F = ma$ →

Resistive Forces

$$D - 10500 - 3000 - 1500 = 80000\,a$$

$$\therefore \quad D - 15000 = 80000\,a$$

So if $D < 15000$ then $a < 0$ so will be decelerating

(ii) If $D = 35000$ N

$$35000 - 15000 = 80000\,a$$

$$\therefore \quad a = \frac{20000}{80000} = \frac{1}{4} = 0.25 \ \text{m/s}^2$$

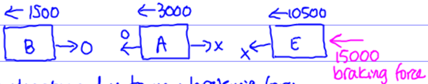
(iii)

If $X = 8500$N when $D = 35000$ N – consider forces on E alone

$$\therefore \quad 35000 - 10500 - 8500 = m \times \frac{1}{4}$$

$$\therefore \quad m = 16000 \times 4 = 64000 \text{ kg}.$$

(iv) the mass of B?



need to find new acceleration due to new braking force

$\therefore \underset{\text{(braking)}}{-15000} \; \underset{\text{(resistive)}}{-15000} = 80000a$ on whole train

$\therefore a = \dfrac{-30000}{80000} = -0.375 \text{ m/s}^2$ expect deceleration because braking

now consider B alone:

$-1500 = m \times (-0.375)$

$\therefore m = \dfrac{1500}{0.375} = 4000 \text{ kg}$

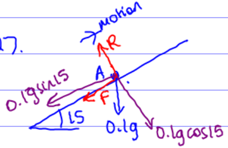
(v) to find new X, look at A say : $m_A = 80000 - 64000 - 4000 = 12000$

$\therefore X - 3000 = 12000 \times (-0.375)$

$X = 3000 - 4500 = -1500 \text{ N}.$

Since we have a negative X, this implies our direction is wrong so X is actually pushing into the back of the engine.

Q7.



$u = 6\,m/s$ (i) $v = u + at$

$t = 1.5$ seconds $0 = 6 + 1.5a$

$v = 0$ $\therefore\ a = \frac{-6}{1.5} = -4\,m/s^2.$

$\therefore\ R = 0.1g\cos 15$

and $-F - 0.1g\sin 15 = 0.1a$ $(F = ma)$

0.1 kg is mass
0.1g N is weight.

since $F = \mu R$ $\mu = \frac{F}{R} = 0.1546 = 0.155$ (3sf)

(ii) The particle will move back down if the force $0.1g\sin 15$ is $>$ friction.

So, R same, this time F in opposite direction to before if anticipating downwards motion.

$\therefore\ 0.1g\sin 15 > \mu 0.1g\cos 15$

and $\mu < \tan 15$

$0.155 < 0.268$ so moves downwards

(iii) Speed back through A.

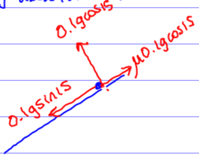
distance up to point where particle stops $s = \left(\frac{u+v}{2}\right)t = \frac{1}{2}(6+0) \times 1.5$

$\therefore \; s = 4.5$ metres

so coming back down, the component of gravity used for acceleration

$0.1g \sin 15 - \mu 0.1g \cos 15 = 0.1a$

$\therefore \; a = 1.0728 \; m/s^2.$



so for $v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times (1.0728) \times 4.5$

$\therefore \; v = 3.108$ m/s

$v = 3.11$ m/s on way down through A.