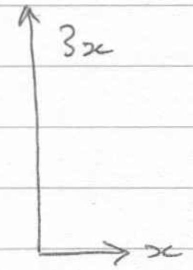
M1. June 2009

1. (i)



If resultant has magnitude 6N

then $(3x)^2 + (x)^2 = 36$

$\therefore 9x^2 + x^2 = 36$

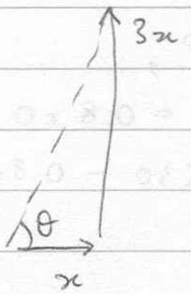
$\therefore 10x^2 = 36$

$x^2 = 3.6$

$x = \sqrt{3.6} = 1.897$

$\therefore x = 1.90$ (3 sf)

(ii) angle :



$\tan\theta = \frac{3x}{x} = 3$

$\therefore \theta = \tan^{-1}(3) = 71.565^\circ$

$\therefore \theta = 71.6^\circ$ (3sf)

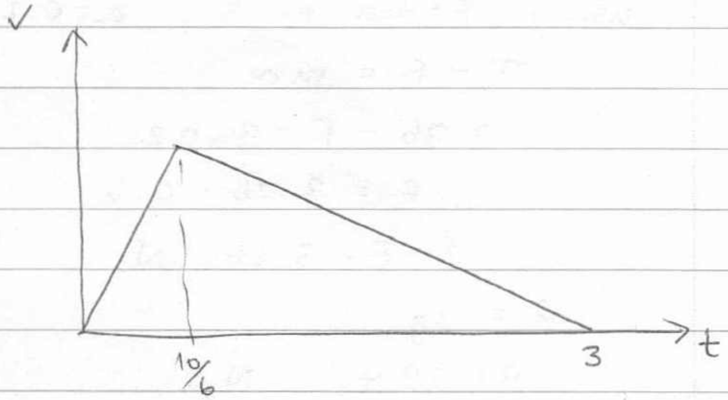
2. (i) $u = 0$

$v =$

$a =$

$s =$

$t =$



(ii) since $s = 6$ metres

distance = area under graph

$\therefore \frac{1}{2} \times 3 \times V = 6$

$\therefore V = 4$ m/s

(iii) $a = 2.4\ ms^{-2}$

$v = u + at$

$4 = 0 + 2.4t$

$t = \frac{4}{2.4} = \frac{10}{6}$ secs.

$\therefore 3 - \frac{10}{6} = \frac{8}{6}$ for t_2

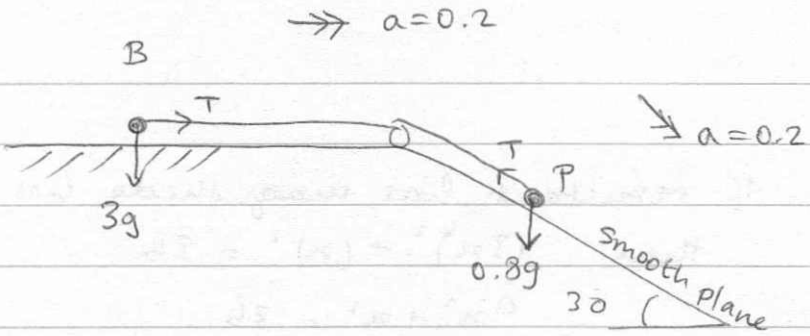
$v = u + at$

$0 = 4 + (a)(t)$

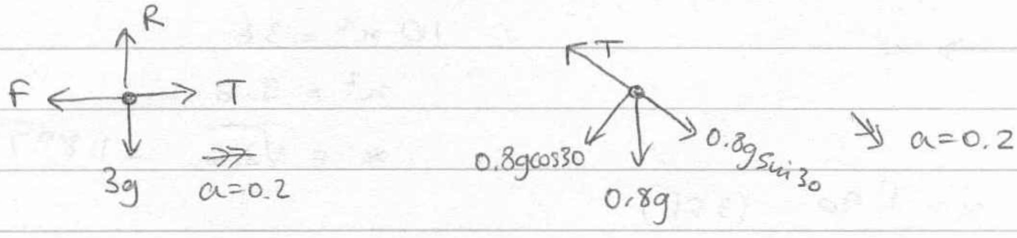
$\therefore 0 = 4 + a\left(\frac{8}{6}\right)$

$\therefore a = -\frac{4 \times 6}{8} = -3\ ms^{-2}$

3.



(i)



B. P

using $F=ma$ on P,

$0.8g\sin 30 - T = 0.8 \times 0.2$

$\therefore T = 0.8g\sin 30 - 0.8 \times 0.2$

$\therefore T = 3.76$ N

(ii) using $F=ma$ on B, $a=0.2$ and $T=3.76$

$T - F = ma$

$\therefore 3.76 - F = 3 \times 0.2$

$\therefore F = 3.76 - 0.6$

$\therefore F = 3.16$ N

$R = 3g$

$\therefore R = 29.4$ N

using $F \leq \mu R$

$\mu = \dfrac{F}{R} = \dfrac{3.16}{29.4} = 0.10748\ldots$

$\therefore \mu = 0.107$ (3sf)

4. (i) $u = 7 \uparrow$ $v^2 = u^2 + 2as$

$v = ?$ $\therefore v^2 = 49 + 2(-9.8)(2.1)$

$a = -9.8 \uparrow$ $\therefore v^2 = 7.84$

$s = 2.1$ $\therefore v = 2.8 \text{ ms}^{-1}$

$t = x$

(ii) greatest height when $v = 0$

$\therefore v^2 = u^2 + 2as$

$0 = 49 + 2(-9.8)s$

$\therefore s = \frac{49}{19.6} = 2.5$

$\therefore s = 2.5$ metres

(iii) travelling downwards with speed 5.7 ms^{-1}

$s = ut + \frac{1}{2}at^2$ $\times$ $v = u + at$

$u = 7 \uparrow$ $u = 0$ at maximum height

$v = -5.7 \ (\downarrow)$ $v = 5.7$

$a = -9.8 \uparrow$ $g = a = 9.8$

going up is +ve $\therefore 5.7 = 0 + 9.8t$

$v = u + at$ $\therefore t = \frac{5.7}{9.8} = 0.5816\ldots$

$0 = 7 - 9.8t$

$t = \frac{7}{9.8} = 0.7142\ldots$

$\therefore$ total time $= 0.5816 + 0.7142 = 1.2959\ldots$

$\therefore$ total time taken $= 1.30 \text{ s}$ (3sf)

$\leftarrow$ positive direction.

5. (i) before

$$Q \xrightarrow{} 0 \qquad \xleftarrow{6\,m/s} P$$

m $\qquad$ 0.5

after

$\xleftarrow{} (v+1)$ $\qquad$ $\xleftarrow{} v$

using conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\therefore \quad 0 + (0.5)(6) = (m)(v+1) + (0.5)(v)$$

$$\therefore \quad mv + m + 0.5v = 3$$

$$\therefore \quad v(m+0.5) + m = 3$$

$$\therefore \quad v(m+0.5) = -m+3 \qquad (1)$$

(ii) before

$$Q \xrightarrow{} 4 \qquad \xleftarrow{} 2 \; P$$

m $\qquad$ 0.5 $\qquad$ $\longrightarrow$ positive direction

after $\longrightarrow v$ $\qquad$ $\longrightarrow (v+1)$

$$\therefore \quad 4m + (-2)(0.5) = mv + (0.5)(v+1)$$

$$\therefore \quad 4m - 1.0 = mv + 0.5v + 0.5$$

$$\therefore \quad 4m - 1.5 = v(m+0.5)$$

$$\therefore \quad v(m+0.5) = 4m - 1.5 \qquad (2)$$

(iii) if (1) = (2) then $-m+3 = 4m - 1.5$

$$\therefore \quad 5m = 4.5$$

$$\therefore \quad m = 0.9 \text{ kg}$$

$$\therefore \quad v(0.9+0.5) = -0.9+3$$

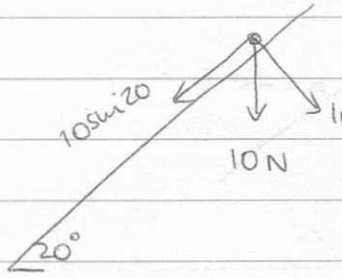
$$\therefore \quad v = \frac{2.1}{1.4} = \frac{3}{2} = 1.5 \text{ ms}^{-1}$$

$$\therefore \quad v = 1.5 \text{ ms}^{-1}$$

Q6.

(i)

(a) $\nearrow a = 0$ constant speed



parallel to plane

resolved component $= 10\sin 20$

perpendicular to plane

resolved component $= 10\cos 20$

(b) using $F = ma$ down plane $a = 0$ implies $F = 10\sin 20$ (up the plane) for frictional force

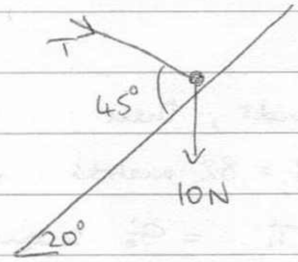
$R = 10\cos 20$

$\therefore$ $F = \mu R$ gives $\mu = \dfrac{F}{R} = \dfrac{10\sin 20}{10\cos 20} = \tan 20$

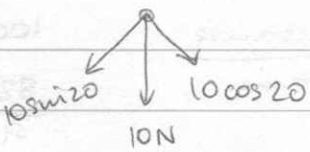
$\therefore \mu = 0.3639\ldots$

$\therefore \mu = 0.364$ (3 sf)

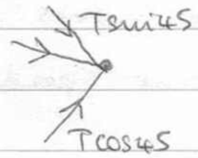
(ii)

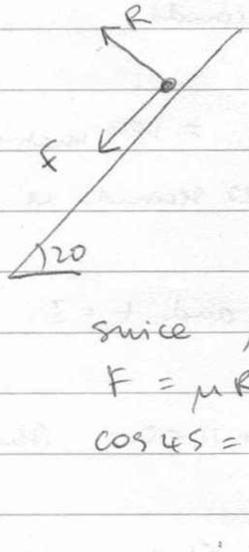


Resolving 10N



Resolving T





given that frictional force acts down plane when B in equilibrium

Using $F = ma$ (up slope) $T\cos 45 - 10\sin 20 - F = 0$

Perpendicular forces $R = 10\cos 20 + T\sin 45$

since $\mu = 0.3639\ldots$ and limiting friction applies

$F = \mu R$ $\therefore$ $T\cos 45 - 10\sin 20 = \mu\,(10\cos 20 + T\sin 45)$

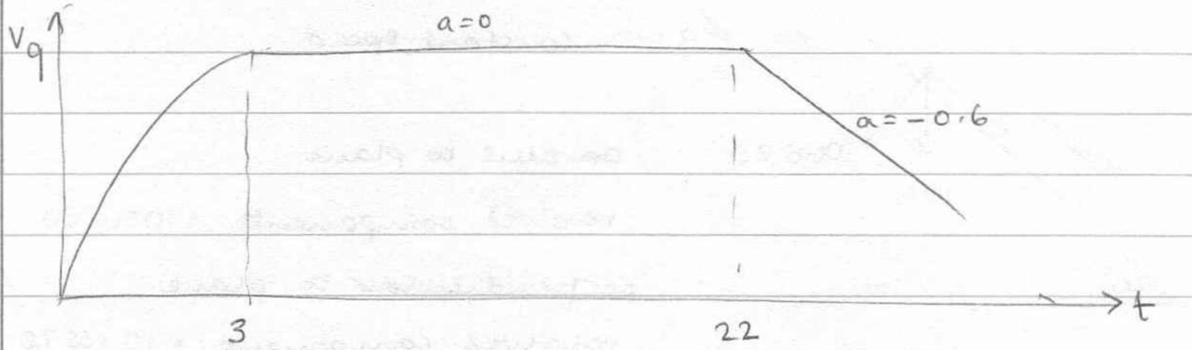
$\cos 45 = \sin 45 = \dfrac{1}{\sqrt{2}}$ $T\left(\cos 45 - \mu\sin 45\right) = 10\sin 20 + \mu\, 10\cos 20$

$\therefore T\left(0.4497\ldots\right) = 6.8397\ldots$

$\therefore T = 15.20\ldots$

$\therefore T = 15.2\ \text{N}$ (3sf)

Q7.



(i) for $0 \le t \le 3$

$v = 6t - t^2$

$\therefore \dfrac{dv}{dt} = acc = 6 - 2t$

(ii) for $0 \le t \le 3$

$x = \dfrac{6t^2}{2} - \dfrac{t^3}{3} + c$ since $x = 0$ when $t = 0$, at rest at start, then $c = 0$

for $t = 3$ then $x = 3 \times 9 - 9 = 18$

$\therefore x = 18$ metres

(iii) for 100 metres, first 18 metres in 3 seconds, then

constant speed $= \dfrac{\text{distance}}{\text{time}}$ $\quad 100 - 18 = 82$ metres

$\dfrac{82}{9} = T_1 = 9\frac{1}{9}$ seconds

$\therefore$ time to 100 metres is $3 + 9\frac{1}{9} = 12\frac{1}{9}$ seconds.

(iv) distance in first 22 seconds $= 18 + 9 \times 19 = 189$ metres.

so 11 metres run during period when $t > 22$ seconds ie period of deceleration

$S = vt - \frac{1}{2}at^2$ where $v = 9$ and $a = -0.6$

$\therefore S = 11 = 9t - 0.3t^2$ so $0.3t^2 - 9t + 11 = 0$ (then $\times 10$)

$\therefore 3t^2 - 90t + 110 = 0$

$\therefore t = \dfrac{+90 \pm \sqrt{(90)^2 - 4(3)(+110)}}{6} = \dfrac{+90 \pm 82.34}{6}$ (t not negative) [cannot be $\frac{172}{6}$ too large]

$\therefore t = \dfrac{7.659}{6} = 1.277$ seconds $= 1.28$ seconds.

Total time $= 22 + 1.28 = 23.3$ seconds.