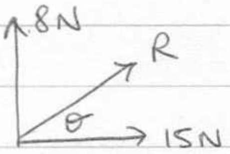
M1 June 2011.

1.



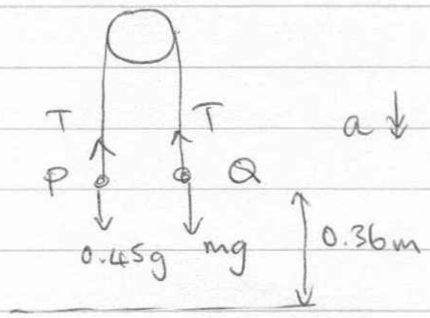
$R^2 = 8^2 + 15^2$ $\qquad \theta = \tan^{-1}\left(\frac{8}{15}\right) = 28.07..$

$\therefore R = \sqrt{289}$

$\therefore R = 17N.$ $\qquad \therefore \theta = 28.1^\circ$ (3sf)

2. $\qquad a = 0.98 \ m/s^2$

(i) & (ii)



$mg - T = ma$

$T - 0.45g = 0.45a$

$\therefore mg - 0.45g = 0.98m + 0.45 \times 0.98$

$\therefore m(g - 0.98) = 0.45(g + 0.98)$

$\therefore m = \frac{4.851}{8.82} = 0.55$

$\therefore m = 0.55$ kg

and $T = 4.851$ N

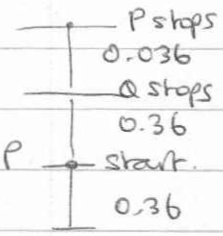
$\therefore T = 4.85$ N (3sf)

(iii) $v^2 = u^2 + 2as$

$\therefore v^2 = 0 + 2 \times 0.98 \times 0.36$

$\therefore v = 0.84$ m/s Q strikes the floor with this speed.

(iv) P continues to rise after Q stops on the floor but the acceleration changes to $-9.8 \ m/s^2$



$u = 0.84$ $\qquad v^2 = u^2 + 2as$

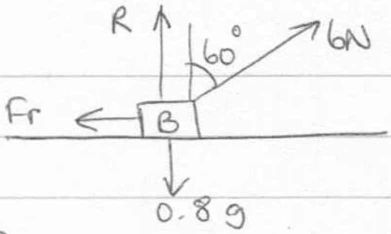
$v = 0$ $\qquad 0 = 0.84^2 + 2 \times (-9.8) \times s$

$a = -9.8$ $\qquad \therefore s = \frac{0.84^2}{19.6} = 0.036$ metres.

$s = ?$

So P rises 0.36 m + 0.036 m + 0.36 m in total above the floor level = 0.756 metres.

3.



$\mu = 0.2$

$\uparrow 6\cos 60$ $\rightarrow 6\sin 60$ Resolving 6N force

(i) $R + 6\cos 60 = 0.8g$ resolving vertically

$\therefore R = 0.8g - 6\cos 60$

$\therefore R = 4.84$ N

(ii) using "$F = ma$" $6\sin 60 - fr = 0.8a$

since $F_r = \mu R$ then $fr = 0.2 \times 4.84 = 0.968$ N

$\therefore a = \dfrac{6\sin 60 - 0.968}{0.8} = 5.285\ldots$

$\therefore a = 5.29$ m/s^2 (3sf)

(iii) when velocity 4.9 m/s 6N force removed and decelerates to rest.

using "$f = ma$" $-fr = 0.8a$

but $fr = 0.2R$ where $R = 0.8g$

$\therefore fr = 0.2 \times 0.8g = 1.568$ N

$\therefore a = -1.96$ m/s^2

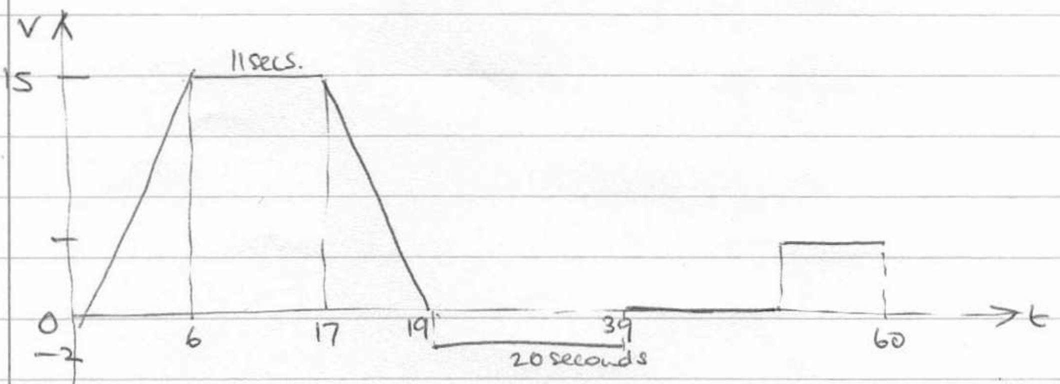
$\therefore v = u + at$

$0 = 4.9 - 1.96t$

$\therefore t = \dfrac{4.9}{1.96} = 2.5$ seconds

$\therefore$ takes $2\frac{1}{2}$ seconds to stop motion

4.



(i) initial acceleration $v = u + at$ $\therefore 15 = 0 + 6a$ $\therefore a = \frac{15}{6}$ m/s^2

deceleration of car $v = u + at$ $0 = 15 + 2a$ $\therefore a = -\frac{15}{2}$ m/s^2

(ii) the car travels a distance equal to the area of the trapezium

$s = \frac{1}{2}(11 + 19) \times 15 = 225$ metres.

(iii) walks $s = ut = (-2) \times 20 = -40$

$\therefore$ walks 40 metres

jogs $s = ut = (5) \times t$

but same distance back as walking ie 40m

$\therefore 40 = 5t$ so $t = 8$ for jogging.

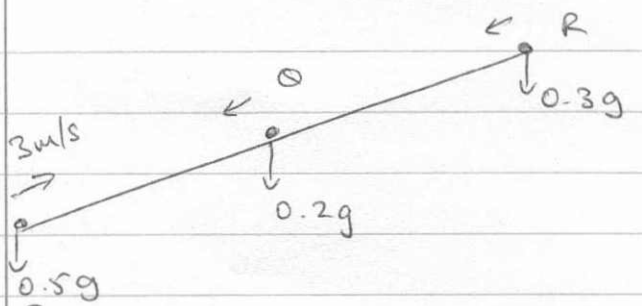
So in the shop

60 (total) − jog (8 seconds) − walk (20 seconds) − car (19 s)

= 13 seconds.

5.

(i)



$a = 2.5\ m/s^2$ down plane.

0.4 seconds after start

for P : $v = u + at$

$v = 3 + (-2.5) \times 0.4$

$\therefore v = 2\ m/s$

[when going up $a = -2.5$
down $a = +2.5$]

for Q : $v = u + at$

$v = 0 + 2.5 \times 0.4$

$v = 1\ m/s$

using conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.5(2) + 0.2(-1) = 0.5v_1 + 0.2(3.2)$

$\therefore 0.5v_1 = 0.16$

$\therefore v_1 = 0.32$

m = 0.5 P: $u \rightarrow 2$, $v_1 \rightarrow$

m = 0.2 Q: $u \leftarrow -1$, $v \rightarrow 3.2$

(up the plane) ⇒

So P travels speed 0.32 m/s up the plane after collision

(ii) Q travels for 0.6 seconds and coalesces with R

for Q : $v = u + at$

$v = 3.2 + (-2.5) \times 0.6$

$v = 1.7\ m/s$

for R (it travels total 1 sec)

$v = 0 + (+2.5) \times 1$

$v = 2.5\ m/s$

Conservation of momentum

$0.2(1.7) + 0.3(-2.5) = 0.5(-v)$

$\therefore 0.5v = 0.41$

$\therefore v = 0.82\ m/s$

Q m = 0.2: $u \rightarrow 1.7$

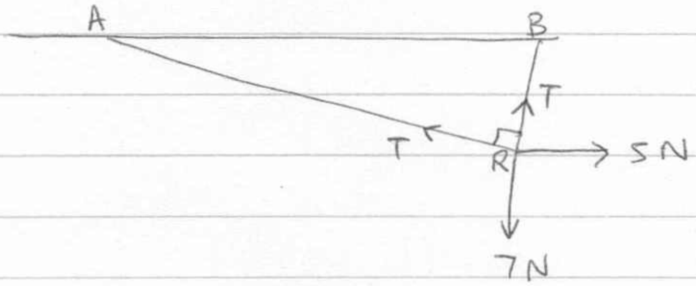
R m = 0.3: $u \leftarrow 2.5$ (down)

QR m = 0.5

$\leftarrow v$ assume going down

$\therefore$ Q + R move down plane with speed 0.82 m/s

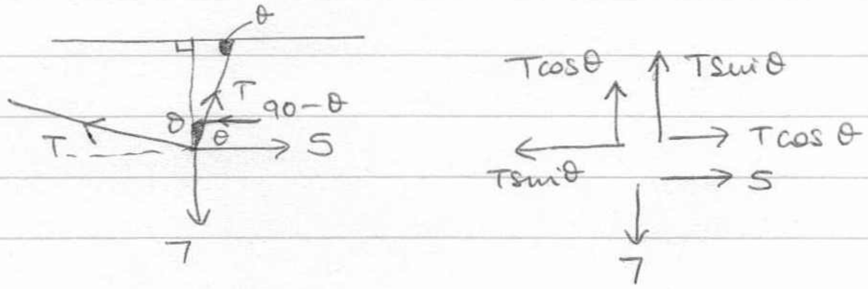
6.



same tension in AR and RB as the same string.

(i) ring is threaded onto one string and hangs on the string so same tension: we are told it is smooth ring so there will be no friction affecting the string.

(ii)



$\therefore$ $T\sin\theta = T\cos\theta + 5$ (horizontally)

& $T\cos\theta + T\sin\theta = 7$ (vertically)

(iii) $\therefore$ $T\cos\theta + T\cos\theta + 5 = 7$

$\therefore$ $2T\cos\theta = 2$

$T\cos\theta = 1$

$\therefore$ $T\sin\theta = 1 + 5 = 6$

$\left.\begin{matrix} T\cos\theta = 1 \\ T\sin\theta = 6 \end{matrix}\right\} T^2 = 1^2 + 6^2$

$\therefore$ $\dfrac{T\sin\theta}{T\cos\theta} = \dfrac{6}{1}$

$\therefore$ $\tan\theta = 6$

$\therefore$ $\theta = 80.5^\circ$ (3SF)

$\therefore$ $T = 6.08$ N (3sf)

7. $x = 0.1t^3 - 0.3t^2 + 0.2t$

(i) $v = \dfrac{dx}{dt} = 0.3t^2 - 0.6t + 0.2$

$a = \dfrac{dv}{dt} = 0.6t - 0.6$

(ii) When $a = 0$ $\quad 0.6t - 0.6 = 0$

$\therefore t = 1$

$\therefore \quad x = 0.1 \times 1^3 - 0.3 \times 1^2 + 0.2 \times 1 = 0$

$\therefore$ displacement at time $t = 1$ is 0, so P is at O.

(iii) When P is stationary then $\frac{dx}{dt} = 0$ since $v = 0$

$\therefore \quad 0.3t^2 - 0.6t + 0.2 = 0$

$3t^2 - 6t + 2 = 0$

$\therefore t = \frac{+6 \pm \sqrt{36 - 4 \times 2 \times 3}}{6} = \frac{+6 \pm \sqrt{12}}{6}$

$\therefore t = 1.57735\ldots$ or $t = 0.4226\ldots$

$\therefore t = 1.58$ seconds or $t = 0.423$ seconds (3sf)

(iv) for Q $\quad v = 0.2t^2 - 0.4$

Q and P meet when their displacements are the same

so for Q $\quad x = \frac{0.2t^3}{3} - 0.4t + c$

Since Q leaves O $x = 0$ when $t = 0$ implies $c = 0$

(Q) $\quad \therefore \frac{0.2t^3}{3} - 0.4t = 0.1t^3 - 0.3t^2 + 0.2t$ (P)

$\therefore \quad 2t^3 - 12t = 3t^3 - 9t^2 + 6t$

$\therefore \quad t^3 - 9t^2 + 18t = 0$

$(t - 3)(t - 6) = 0.$

$\therefore t = 3$ or $t = 6$ seconds.

Since we want the first collision this occurs at $t = 3$ seconds.