Solutions to January 2010

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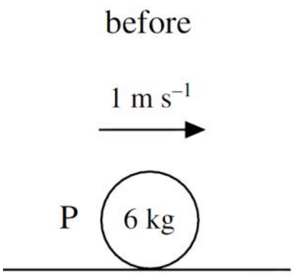
<td>1(a)</td>
<td>


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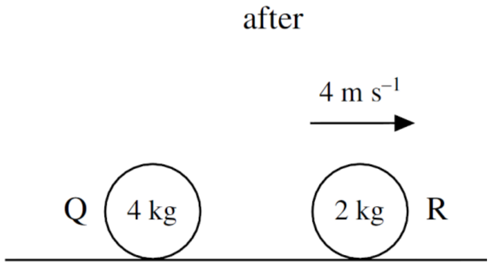

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<td>i.</td>
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$m_1 = 4$
$u_1 = 1$
$m_2 = 2$
$v_1 = ?$
$v_2 = 4$

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This is the reverse of two particles coalescing.
Using Conservation of momentum
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
$6(1) = 4v_Q + 2(4)$
$\therefore 4v_Q = -2$
$\therefore v_Q = -0.5$
Direction is opposite to P's original motion

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<td>ii.</td>
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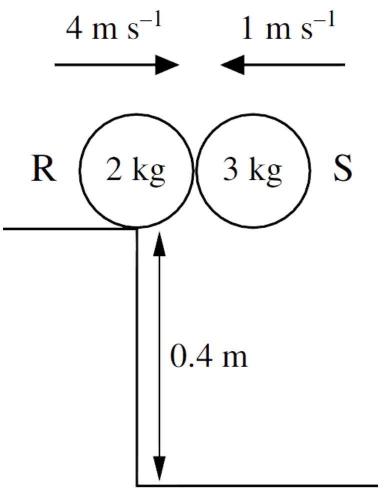

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Coefficient of restitution *e*=0.1
$e = \dfrac{v_2 - v_1}{u_1 - u_2} = \dfrac{v_S - v_R}{u_R - u_S}$
$\therefore 0.1(u_R - u_S) = v_S - v_R$
$\therefore 0.1(4 - -1) = v_S - v_R$
$\therefore v_S - v_R = 0.5$
Using Conservation of momentum
$m_Ru_R + m_Su_S = m_Rv_R + m_Sv_S$
$2(4) + 3(-1) = 2v_R + 3v_S$
$\therefore 2v_R + 3v_S = 5$
Solving both equations
$v_R = 0.7$
$v_S 1.2$

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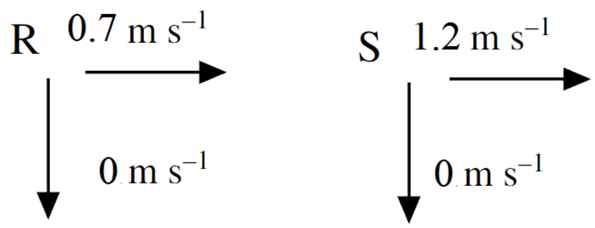
To find the distance horizontally between particles when they land, split the calculations into SUVAT in H and V. Note that for V, a=9.8 and for H, a=0. Since both particles only have initial velocity in H, they land at the same t for V.



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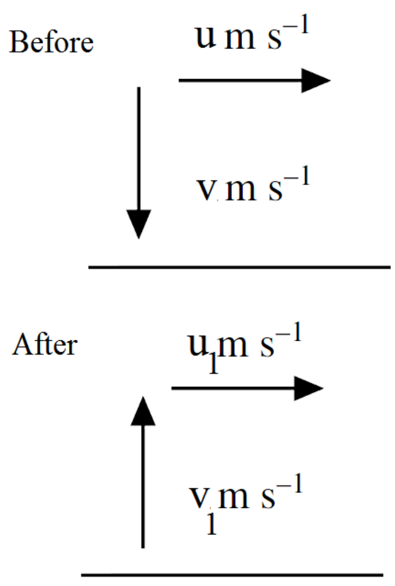
iii.	Vertically for R & S $s = 0.4$ $u = 0$ $v = ?$ $a = 9.8$ $s = ut + 0.5at^2$ $0.4 = 0 + 4.9t^2$ $\therefore t = \frac{2}{7}$	Horizontally for R $s = ?$ $u = 0.7$ $a = 0$ $s = ut + 0.5at^2$ $s = 0.7 \times \frac{2}{7} + 0$ $\therefore s = 0.2$	Horizontally for S $s = ?$ $u = 1.2$ $a = 0$ $s = ut + 0.5at^2$ $s = 1.2 \times \frac{2}{7} + 0$ $\therefore s = \frac{12}{35}$
	Distance apart $= \frac{12}{35} - 0.2 = \frac{1}{7} = 0.143$ metres		
(b)	 Before impact, velocities are u and v After impact let velocities be u_1 and v_1 Coefficient of restitution is *e* Velocity perpendicular to plane is affected by coefficient of restitution, but velocity parallel to plane remains unchanged	$e = \frac{v_1}{v} \Rightarrow v_1 = ev$ $u_1 = u$ Before impact $k.e. = \frac{1}{2}mu^2 + \frac{1}{2}mv^2$ After impact $k.e. = \frac{1}{2}mu_1^{\ 2} + \frac{1}{2}mv_1^{\ 2}$ $k.e. = \frac{1}{2}mu^2 + \frac{1}{2}m(ev)^2$ Loss in mechanical energy = ke (before impact) – ke (after impact) $= \frac{1}{2}m\left(u^2 + v^2\right) - \frac{1}{2}m\left(u^2 + (ev)^2\right)$ $= \frac{1}{2}m\left(v^2 - e^2v^2\right)$ $= \frac{1}{2}mv^2\left(1 - e^2\right)$	

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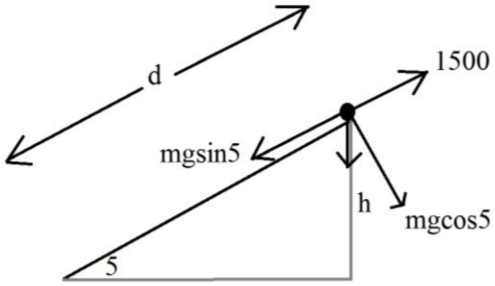
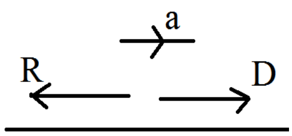
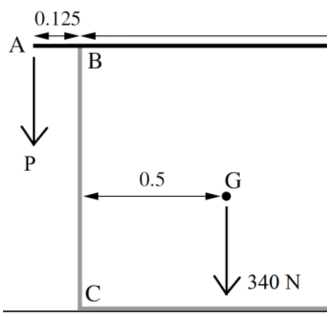
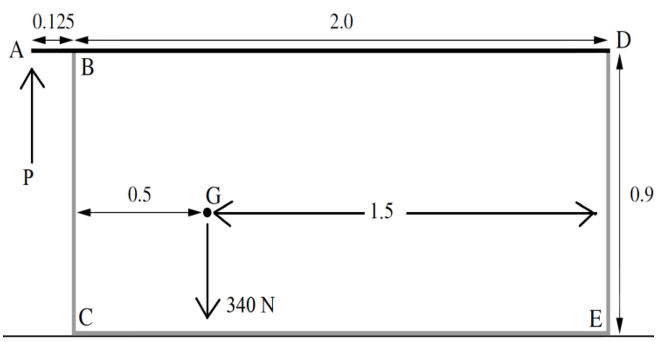
2i.	Work done = Fd Work done against gravity i.e. gravitational potential energy = *mgh* Power $= Fv = F\frac{d}{t} = \frac{Fd}{t} = \frac{work\ done}{time}$	
	$m = 1200$ $h = 60$ $wdR = 1.8 \times 10^6$ $u = v$ $t = 120$	Total work done overcomes gravity and resistances $= 1200 \times 9.8 \times 60 + 1800000$ $= 2505600$ $\therefore P = \frac{2505600}{120} = 20880$ Power=20.9 KW
	You could do this using conservation of energy *wd by driving force – mgh – wd by resistances = change in k.e.* $Dx - mgh - 1800000 = \frac{1}{2}m(v^2 - u^2)$ $\therefore Dx = 600 \times 0 + mgh + 1800000$ $\therefore P = \frac{Dx}{120} = \frac{2505600}{120} = 20880$	
ii.	$v = 18$ $P = 13500$	There is no acceleration when at steady speed so the driving force D equals the resistive forces R $P = Fv$ $\therefore F = \frac{13500}{18} = 750$ Where F is the overall forward (i.e. in this case driving) force. So the resistive force is also 750N. Over 200 metres, the work done against resistance $= Fd = 750 \times 200 = 150000$ Joules
iii.	$u = 18, v = 9$ $P = 0$ resistance $= 1500$ $\sin 5 = \frac{h}{d}$ $\therefore h = d \sin 5$	

	Energy methods means work done, *mgh*, and change in k.e. Total work done =change in k.e	$gpe = mgh = mgd\sin 5$ wd by resistance $s = 1500d$ $\therefore mgd\sin 5 - 1500d = \frac{1}{2}m(v^2 - u^2)$ $\therefore d(1200\times 9.8\sin 5 - 1500) = 600(9^2 - 18^2)$ $\therefore d = \frac{145800}{475.048....} = 306.91...$ $\therefore d = 307$
iv.	Using P, v, a and R  	$P = Dv$ $F = ma \Rightarrow D - R = ma$ $\therefore D = R + 1200a$ $\therefore P = (R + 1200a)v$ If P and R are constant then *av* must be constant If a=0 then P and R constant is true If a≠0 then as v is not constant a cannot be constant

3i.	 	If on the point of tipping over corner C due to P acting at A downwards, then Moments about C $P \times 0.125 = 340 \times 0.5$ $\therefore P = 1360N$
	 	If on the point of tipping over corner E due to P acting at A upwards, then Moments about E $P \times 2.125 = 340 \times 1.5$ $\therefore P = 240N$

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<td>ii.</td>
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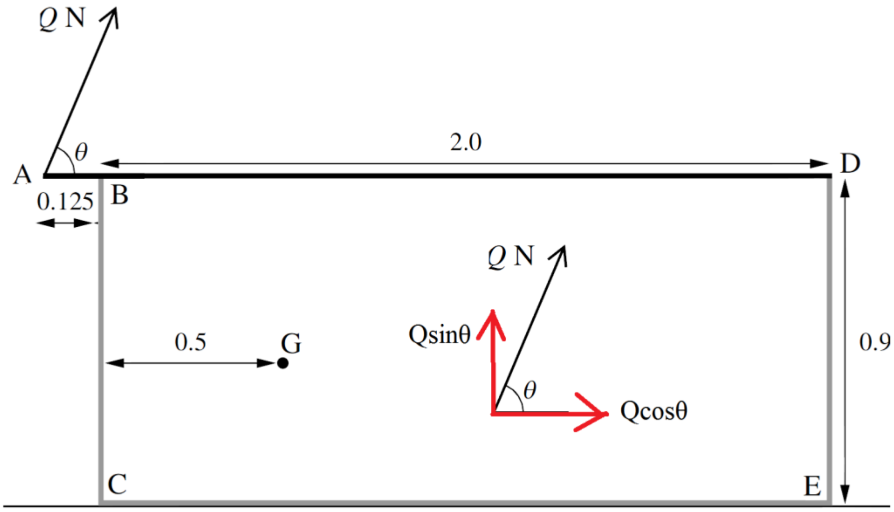

Resolve Q into horizontal and vertical component
Moments about E clockwise for components of Q only

$$= Q\sin\theta \times 2.125 + Q\cos\theta \times 0.9$$

$$= \frac{12}{13}Q \times 2.125 + \frac{5}{13}Q \times 0.9$$

$$= \frac{30}{13}Q$$
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<td>iii.</td>
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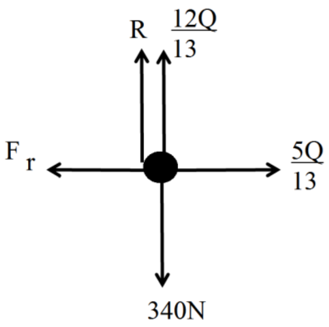

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If on the point of tipping with force Q then
Moments about E for Q and weight

$$\frac{30}{13}Q = 340 \times 1.5$$

$$\therefore Q = 221N$$

Resolving horizontally

$$F_r = \frac{5}{13}Q = 221 \times \frac{5}{13} = 85$$

Resolving vertically

$$R + \frac{12}{13}Q = 340$$

$$\therefore R = 340 - \frac{12}{13}Q = 136$$

Using limiting friction $F_r \le \mu R$

$$\mu \ge \frac{85}{136}$$

$$\therefore \mu \ge \frac{5}{8}$$
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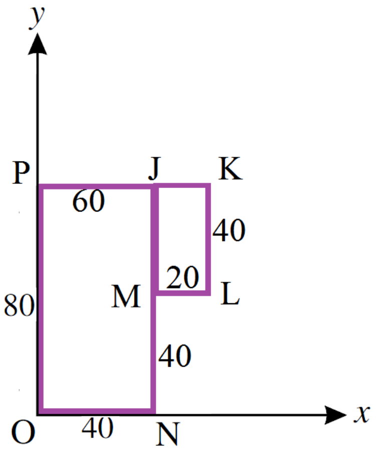
4i.



Area	Mass	Cofm x	Cofm y
PONJ	3200	20	40
JMLK	800	50	60
total	4000		

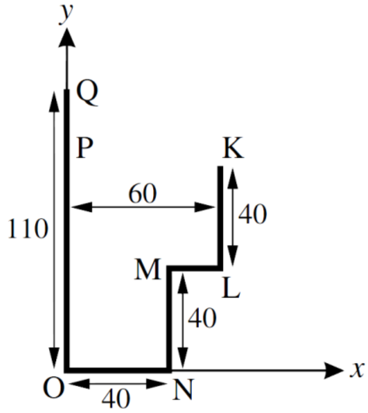
$$4000\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 3200\begin{pmatrix}20\\ 40\end{pmatrix} + 800\begin{pmatrix}40\\ 60\end{pmatrix}$$

$$\therefore 4000\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}104000\\ 176000\end{pmatrix}$$

$$\therefore \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}26\\ 44\end{pmatrix}$$

$$c.m. = (26,44)$$

ii.



Area	Mass	Cofm x	Cofm y
OQ	110	0	55
ON	40	20	0
NM	40	40	20
ML	20	50	40
LK	40	60	60
	250		

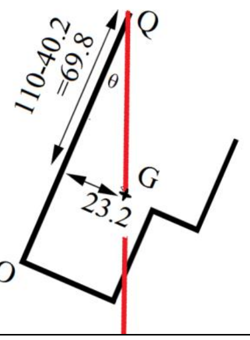
$$250\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 110\begin{pmatrix}0\\ 55\end{pmatrix} + 40\begin{pmatrix}20\\ 0\end{pmatrix} + 40\begin{pmatrix}40\\ 20\end{pmatrix} + 20\begin{pmatrix}50\\ 40\end{pmatrix} + 40\begin{pmatrix}60\\ 60\end{pmatrix}$$

$$\therefore 250\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}5800\\ 10050\end{pmatrix}$$

$$\therefore \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}23.2\\ 40.2\end{pmatrix}$$

$$c.m. = (23.2,40.2)$$

iii.

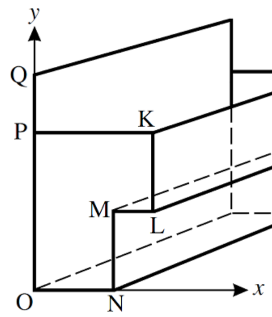


The centre of mass will hang directly beneath point Q

$$\tan\theta = \frac{23.2}{69.8}$$

$$\theta = 18.4°$$

iv.



The ends have the same centre of mass as the coordinates calculated in i.
The sides have the same centre of mass as the coordinates calculated in ii.
Amalgamating this information,

Part	Mass	Cmx	Cmy	Cmz
End1	1.5	26	44	0
End2	1.5	26	44	L
sides	7	23.2	40.2	L/2
Total	10			

Ignoring the z coordinates

$$10\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=1.5\begin{pmatrix}26\\44\end{pmatrix}+1.5\begin{pmatrix}26\\44\end{pmatrix}+7\begin{pmatrix}23.2\\40.2\end{pmatrix}$$

$$\therefore 10\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}240.4\\413.4\end{pmatrix}$$

$$\therefore \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}24.04\\41.34\end{pmatrix}$$

$$c.m.=(24.0,41.3)$$