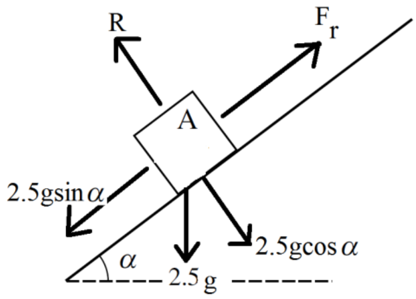
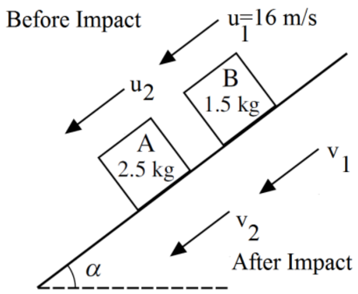
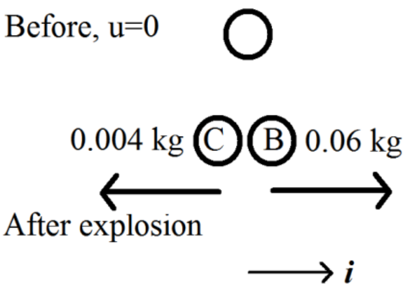
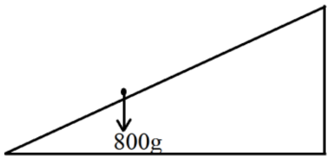
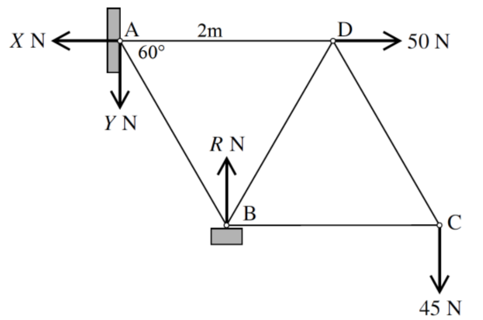
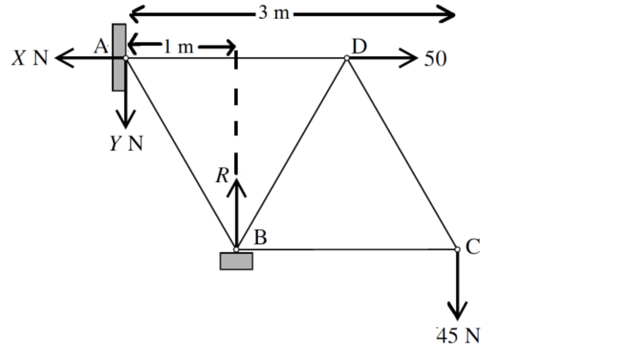
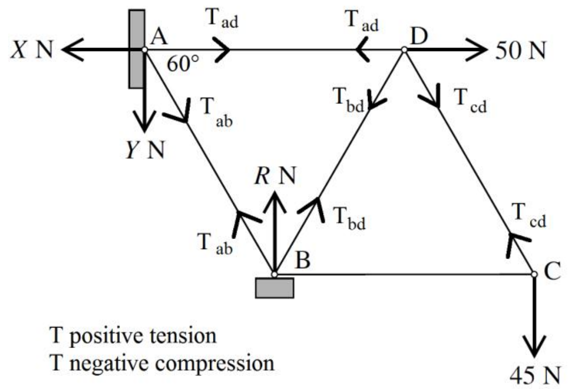
4762 MEI Mechanics 2
Solutions to January 2011

1 i.	 $\cos\alpha = 0.8 = \frac{4}{5}$ $\therefore \sin\alpha = 0.6 = \frac{3}{5}$ Since right angle triangle has sides 3,4,5 in length	Resolving the weight into components parallel and perpendicular to the plane Perpendicular to the plane $R = 2.5g\cos\alpha$ Limiting friction base on $F_{r\max} \le \mu R$ $\mu = 0.85$ $\therefore F_{r\max} \le 0.85 \times 2.5g \times 0.8$ $\therefore F_{r\max} = 1.7g = 16.66N$ If there is motion then $2.5g\sin\alpha \ge F_r$ $2.5g\sin\alpha = 1.5g < 1.7g$ Since the weight component < Fr_{max} there is no motion, i.e. object remains at rest
ii.	 Coefficient of restitution on impact between objects *e*=0.4	$B:$ $m_1 = 1.5$, $u_1 = 16$, v_1 and $A:$ $m_2 = 2.5$, $u_2 = 0$, v_2 Conservation of momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ $1.5(16) + 2.5(0) = 1.5v_1 + 2.5v_2$ $\therefore 1.5v_1 + 2.5v_2 = 24$ $\therefore 3v_1 + 5v_2 = 48$ Using restitution where *e*=0.4 $e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{16 - 0}$ $\therefore 0.4 \times 16 = v_2 - v_1$ $\therefore v_2 = v_1 + 6.4$ Solving for both equations $3v_1 + 5(v_1 + 6.4) = 48$ $\therefore 8v_1 = 48 - 32$ $\therefore v_1 = 2$ $\therefore v_2 = 8.4$
iii.	Impulse $I = Ft = m(v - u)$	$I = 1.5(2 - 16)$ $\therefore I = -21$ B has impulse -21 J going down the slope, i.e. 21 J going up the slope

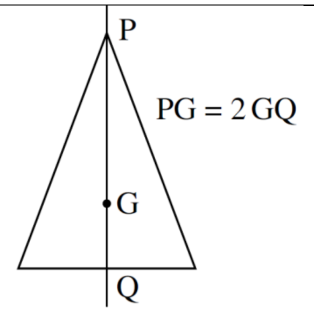
iv.	For A: $u = 8.4$ $v = 0$ $t = ?$ Using F=ma parallel to slope $2.5g\sin\alpha - F_r = 2.5a$ $\therefore a = -\dfrac{0.2g}{2.5} = -0.784$ Using suvat to find t, $v = u + at$ $0 = 8.4 - 0.784t$ $t = \dfrac{8.4}{0.784} = 10.7$	Downwards force $2.5g\sin\alpha - F_r = -0.2g$ Using Impulse $Ft = m(v-u)$ $\therefore -0.2gt = 2.5(0-8.4)$ $\therefore t = \dfrac{2.5\times 8.4}{0.2\times 9.8}$ $\therefore t = 10.7$

2 (a)	If one part moves in direction ***i***, this implies the other part must move in the opposite direction  Remember that in this question, velocities are given in vector form, and at the final answer we must include the direction vector ***i*** Also recall that velocity B is +i, and that for C is -i	Using total kinetic energy is 0.8J $\therefore 0.8 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ $\therefore 0.8 = 0.03v_1^2 + 0.002v_2^2$ $\therefore 30v_1^2 + 2v_2^2 = 800$ $\therefore 15v_1^2 + v_2^2 = 400$ Using conservation of linear momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ $u = 0$ $\therefore 0.06v_1 + 0.004v_2 = 0$ $\therefore 60v_1 + 4v_2 = 0$ $\therefore v_2 = 15v_1$ Solving for both equations $15v_1^2 + (15v_1)^2 = 400$ $\therefore 15v_1^2 + 225{v_1}^2 = 400$ $\therefore 240{v_1}^2 = 400$ $\therefore {v_1}^2 = \dfrac{400}{240}$ $\therefore v_1 = \pm\sqrt{\dfrac{5}{3}} \Rightarrow \underline{v_1} = +\sqrt{\dfrac{5}{3}}\mathbf{i} \Rightarrow \underline{v_2} = -\sqrt{\dfrac{5}{3}}\mathbf{i}$

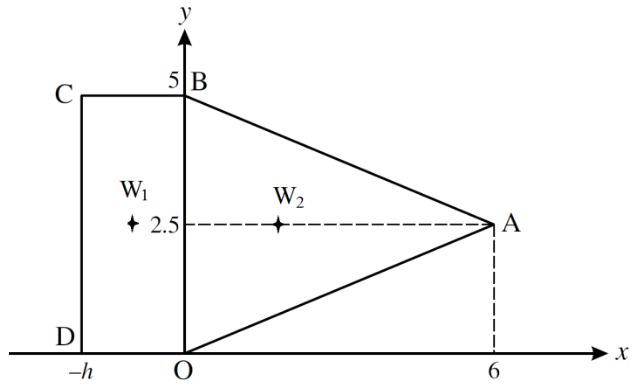
(b) i.	 $u=30$ $v=12$ $h=20$ $D=0$ $R=?$ $m=800$	The vehicle climbing uphill does work taking into account the Work done by gravity (gpe) Work done by resistances Work done = change in kinetic energy $-mgh-wd_R=\frac{1}{2}m(v^2-u^2)$ $\therefore -800\times 9.8\times 20-wd_R=400(144-900)$ $\therefore wd_R=145600$ The work done by the car overcoming resistances is 145600 Joules
ii.	$u=v=18$ $h=20$ $t=25$ $R=750$ $m=800$	Power is the rate of doing work $P=\frac{work\ \ done}{time}$ Total work done is work done overcoming potential energy and the resistances $=mgh+wd_R$ Work done by resistances $=Fs=750s$ $s=ut=18\times 25=450$ $\therefore wd_R=750\times 450$ Thus, the Power of the driving force is $P=\frac{800\times 9.8\times 20+750\times 450}{25}$ $\therefore P=19772$ P=19.8 KW

3i.		Resolving horizontally $X = 50$ Resolving vertically $R = Y + 45$
ii.	Moments about A $R \times 1 = 45 \times 3$ $\therefore R = 135$ $\therefore Y = 90$	
iii.		
iv.	A↕ $T_{ab} \sin 60 + 90 = 0$ $T_{ab} = -60\sqrt{3}$ (compression)	A↔ $T_{ad} + T_{ab} \cos 60 = 50$ $T_{ad} = 50 + 30\sqrt{3}$ (tension)
	C↕ $T_{cd} \sin 60 = 45$ $T_{cd} = 30\sqrt{3}$ (tension)	C↔ $T_{bc} + T_{cd} \cos 60 = 0$ $T_{bc} = -15\sqrt{3}$ (compression)
	D↕ $T_{bd} \sin 60 + T_{cd} \sin 60 = 0$ $T_{bd} = -30\sqrt{3}$ (compression)	D↔ $T_{ad} + T_{bd} \cos 60 = T_{cd} \cos 60 + 50$ $50 + 30\sqrt{3} - 15\sqrt{3} = 15\sqrt{3} + 50$ True: equation balances
v.	C↕ $T_{cd} \sin 60 = 45$ So T_{cd} only depends on external force if angles remain same	C↔ $T_{bc} + T_{cd} \cos 60 = 0$ Shows T_{bc} only depends on T_{cd} if the angles remain the same

4i.



Centre of mass of OAB is (2, 2.5)



ii.

Split into 2 parts assume mass is proportional to area

Part	Mass	C of m x	C of m y
OAB	0.5x6x5=15	2	2.5
OBCD	5h	-h/2	2.5
Total	15+5h		

Centre of mass of combined lamina is

$$(15+5h)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=15\begin{pmatrix}2\\ 2.5\end{pmatrix}+5h\begin{pmatrix}-\frac{h}{2}\\ 2.5\end{pmatrix}$$

$$5(3+h)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}5\left(6-\frac{h^2}{2}\right)\\ 5\times 2.5(3+h)\end{pmatrix}=5\begin{pmatrix}\left(\frac{12-h^2}{2}\right)\\ 2.5(3+h)\end{pmatrix}$$

$$\therefore\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}\left(\frac{12-h^2}{2(3+h)}\right)\\ \frac{2.5(3+h)}{(3+h)}\end{pmatrix}=\begin{pmatrix}\left(\frac{12-h^2}{2(3+h)}\right)\\ 2.5\end{pmatrix}$$

$$c.m.=\left(\frac{12-h^2}{2(3+h)},2.5\right)$$

iii.

The prism will topple about O if

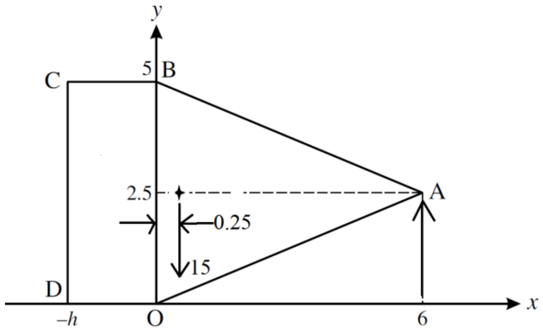
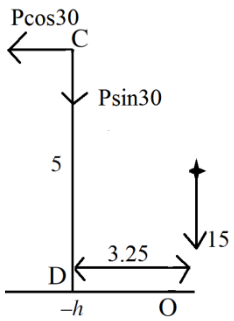
$$W_2\times 2>W_1\times\frac{h}{2}$$

$$\therefore 15\times 2>5h\times\frac{h}{2}$$

$$\therefore h^2<12$$

$$\therefore -2\sqrt{3}<h<2\sqrt{3}$$

However, h is length and h>0 so to topple, $0<h<2\sqrt{3}$

iv.		If on the point of toppling clockwise about O, taking moments about O $15\times0.25 = A\times6$ $\therefore A = 0.625$
v.		If on the point of toppling anticlockwise about D, taking moments about D after resolving force P at C $P\cos30\times5 = 15\times3.25$ $\therefore P = 11.3$